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The Secret of the Circle and the Circle Squared

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The Secret of the Circle

and the

Circle Squared

by

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Where wast thou when I laid the foundations of the earth?
declare, if thou hast understanding.
Who hath laid the measures thereof, if thou knowest?
or who hath stretched the line upon it?

JOB XXXVIII, 4, 5.

The Secret of the Circle

and the

Circle Squared.

ONE paragraph in the pages of the achievements of the Nineteenth Century, affording proof of long and patient thought and investigation, will be that which records the discovery of the true relation of the diameter of the circle to its circumference, and its absolute and exact area. These can as easily be given when the base of the inscribed square is known, as when this is not known, and only its diameter is given.

The difficulties attending, and so far preventing, the solution of this Problem of Problems, have been appreciated in all countries and every age from a remote and early period. It was known very approximately, if not exactly, as we have ample evidence, by the designers, or builders, of the Great Pyramid, supposed to have been built more than 4000 years ago. Whether or no the mathematicians and geometricians of that age had discovered absolute demonstrable truth of this, however, we do not know, for there is nothing on record, nor can we assert it from anything the Pyramid reveals. If these were known, and any proof were in existence, the Greeks would probably have become acquainted with it, for the more we know of their writings the more we see they borrowed much of their

learning from that country. The most curious, analytical, and searching minds, appreciating its importance, which, in many ways, cannot be over-estimated, have been busy with the investigation of this subject, for more than 2000 years. Nor has the consideration of it, in the least, relaxed in the Nineteenth Century, as we know from many attempts made to solve it, and from a comparatively recent paper communicated by Dr. Rutherford to the Royal Society, carrying the calculation, by the present formula, to 200 places of decimals.

Archimedes, certainly the greatest name in Greek science, wrote upon the circle, and quadrature of the parabola, but although he may have convinced, and probably did, himself and others, of the true relationship of the diameter to the circumference, it is certain that he left no proof that we know of; and therefore, perhaps, never found any that was such absolutely, though he may have surmised, from many premisses, and did in fact announce the truth. We do know that he examined this figure from the consideration of a polygon of ninety-six sides, of which more will be said hereafter. The present technical and inadequate formula is not, nor ever has been, altogether convincing or satisfactory, and indeed cannot be, as it is confessedly inaccurate and inexact, and the loss that this occasions cannot at once, or readily, be seen or described. Though apparently under the impression the problem was insoluble, and the true ratio of diameter to circumference could not be ascertained more exactly, or be better expressed, the efforts made go rather to show this was and is not altogether a definite conclusion. The opinion, however, was so authoritative, weighty and universal, that in fact the 'Secret of the Circle,' under which title the writer published a small pamphlet some dozen years ago, but which, as might have been expected, attracted no scientific attention, was looked upon as a dream, mathematically dispelled for ever. When we consider the weight of

scientific authority, of all the schools of learning of the world, and its absolute ultimatum—carrying with it such contradiction, however, as before alluded to—that the squaring of the circle was a practical and absolute impossibility, and therefore anyone attempting it was either a simpleton, or some creature crazed with a hobby, or absolutely mad with presumption and self-conceit, the writer will not perhaps be so much censured for so writing as to keep up an incognito, and publishing under a *nom de plume*, which may now however be acknowledged, and stated was Alick Carrick! To appear, in public, in the face of so universal, abiding, and bitter opposition was thought to be undesirable, and would be now, but the interests of science demand a full and impartial investigation.

It may here be said, to encourage others, and not from any feeling of gratulation, at any rate with not much of this, that this discovery was not made by accident, but from a conviction, from the first, from the very nature and reason of things, that the wrong way had been adopted, and that a right one ought to be and could be found, and that England was the country to find it. The problem was seen to be impossible without an intimate and large knowledge of the absolute values of many different figures of the circle, and some of these are only, approximately, given in the books, when they may easily be found absolutely, by measurement, and from comparison of figure with figure. This in itself is ample proof of the evil of an inexact measure, but when we think of this more than dark shadow, even hurtful blight, cast over many figures of the series of innumerable figures into which the circle may be divided and subdivided, the study of geometry will infallibly lose much of its charm, and when we remember a saying of one of the old Pagan writers, that 'The Deity is always geometrising,' a saying pregnant with meaning, it will generally be admitted this study should be encouraged in every way, and all schools taught to see the

charm and fascination that accompany and follow a more and more intimate acquaintance with the various figures, and so ascertain for themselves the reason for, and root of the various rules. When this search was first prosecuted the writer had no idea of the reason Archimedes might have had in the selection of a polygon of ninety-six sides. It had never occurred to him why this might be, and indeed the construction and examination of such a polygon has only been thought of within the last few months; but it was conceived, from the first, that some segment or segments, and part of a segment, might be ascertained in their value from comparison with some other larger segment or segments. Of course, in saying 'part of a segment,' is meant only the $\frac{1}{2}$ of one. Any other portion must ever, taking it from perimeter to chord, be indefinite, as in a hair's breadth from the line so dividing it from its other half there will necessarily be innumerable degrees. A remarkable fact, illustrating this, will be fully shown in which arithmetic is seen to recognise this, and in a rule for giving the area of larger figures particularly, not only takes cognisance, and specifies the value of, but makes a segment and $\frac{1}{2}$, the corner and foundation stones of the whole calculation and edifice.

And this leads, naturally, to a short consideration of the almost infinite divisibility of matter. The more this is understood and appreciated the more mathematicians—who have only the present inadequate standard, it matters nothing whether it be too short or too long, or both, the truth can never be arrived at by these—will see how unsatisfactory, so unscientific, if we may be forgiven for saying so, such a system is. But the wider and further this divisibility be carried is no difficulty in the computation by the method shown, and which will be proved, for the circle itself is made a measure metre, and measures itself, and all that by possibility it can contain, within a line without breadth.

Suppose a circle be constructed of a thread of finest gossamer;—within the inner and outer perimeters of such thread there is room perhaps for a million lines, showing the boundaries of so many circles, each something larger than the one within. Spectrum analysis discerns $\frac{1}{1000000}$ of a milligramme of natron in a flame. Professor Hertz says there are certainly 3000000000 (three thousand millions) of molecules of matter in an object the size of an ordinary pin's head. Camille Flammarion carries this further and says—whether quite seriously so, or no, is hard to say, for the statement appears in a somewhat fantastic work—there must be 8000 *milliards of milliards* of these (8000000000000000) in an object of such size. But matter that we can see and handle may be dissolved into steam vapour or gas, which are all but other forms of matter proper. But what shall be said about ether? This is, probably, radically different from matter in any known form, and far further divisible in atomic constitution. So also the power or force that causes light to be light, or that becomes light when it has something to act upon, just as, in a way, meteorolites become luminous when they come into contact with a sufficiently dense portion of our atmosphere: and the power of gravitation also acting upon and through ether, for, as Newton says, space must be filled with something however attenuated. Were this not so, his laws would be baseless, and could not exist; and we know they are proved in countless thousand ways.

Now as the circle is to measure the orbit of the spheres and the spheres themselves exactly, it is of the utmost importance it should be exactly known itself, and this is what this paper is written to prove. There is probably, almost certainly, no other way than the one found to the solution. Had there been more than one, the clues and indications occurring, which would have presented themselves in more ways than one, just as the one discovered, as it will be seen, presents itself in several ways, would

have been seen in this long and laborious search of centuries, and have led to the discovery.

The following propositions had all to be carefully considered, and will be proved.

1. A curved line is length without breadth, and,—divisible into innumerable parts or points—is equivalent to a straight one.
2. The same length of line that bounds the circle, may be shown to be the perimeter of an area equal to the area of the inscribed square.
3. The same length of line that bounds the circle may be shown to be the perimeter of the inscribed dodecagon.
4. The inscribed dodecagon is equal to the inscribed square and $\frac{1}{7}$ such square.
5. A segment and $\frac{1}{7}$ a segment, or segments, may be aliquot parts of a straight-sided portion of $\frac{1}{7}$ another segment, or of segments containing them.
6. The base of the inscribed square of the circle and the $\frac{1}{4}$ of such base, multiplied by the same, produce the area of the dodecagon, or square and $\frac{1}{7}$ square—and $\frac{1}{7}$th of such $\frac{1}{7}$ square.
7. The first figure multiplying, as above, sometimes gives the $\frac{1}{7}$th of such $\frac{1}{7}$ square, but when ciphers are not used and the figures are placed below the figures multiplied, the first figure multiplying gives the area of 1 segment and a $\frac{1}{7}$ segment of such 12 segments as constitute the difference between the circle and its dodecagon. The $\frac{1}{7}$th of such $\frac{1}{7}$ square is therefore a multiple of such 1 segment and $\frac{1}{7}$ segment.
8. A figure equal to the dodecagon may be constructed and bounded by a line equal to $\frac{1}{7}$ths of the circumference, and by two straight ones; the one the chord of $\frac{1}{7}$ a segment, and the line dividing such $\frac{1}{7}$ larger segment from its other half.
9. Such chord, as above, and part of the line dividing this segment from its other $\frac{1}{7}$, so far as it reaches to the smaller $\frac{1}{7}$ segment, bound an area equal to the area of $10\frac{1}{7}$ such segments as bound the dodecagon.
10. Such 1 segment and $\frac{1}{7}$ segment are therefore an aliquot part of such straight-sided figure mentioned, and are $\frac{1}{7}$th of an area equal to $10\frac{1}{7}$ such similar segments, because 7 times 1 and $\frac{1}{7}$ are $10\frac{1}{7}$.
11. The 12 segments that bound the dodecagon, or their equivalent, are contained in such $\frac{1}{7}$ segment.
12. The circle contains an area equal to 264 such segments.

Figure 1. Similar segments, otherwise placed, chord to chord, with equal ones leave the same area. As segments 1, 2, 3, 4 are the same, or equivalent to the 4 that bound the inscribed square, the pyriform figure 1 is equal to the inscribed square, and is bounded by a line equal in length to the circumference.

For similar reasons Figure 2 is equal to the dodecagon, and is bounded by a line equal in length to the circumference.*

* Fig. 1. Let the diameter be 1000 this × 500 makes the area of the inscribed square 500000. (Fig. 2.) The area will be 500000 inscribed square and $\frac{1}{2}$ this 250000 = 750000. It would be interesting to know how much less, or more, the present formula will make these.

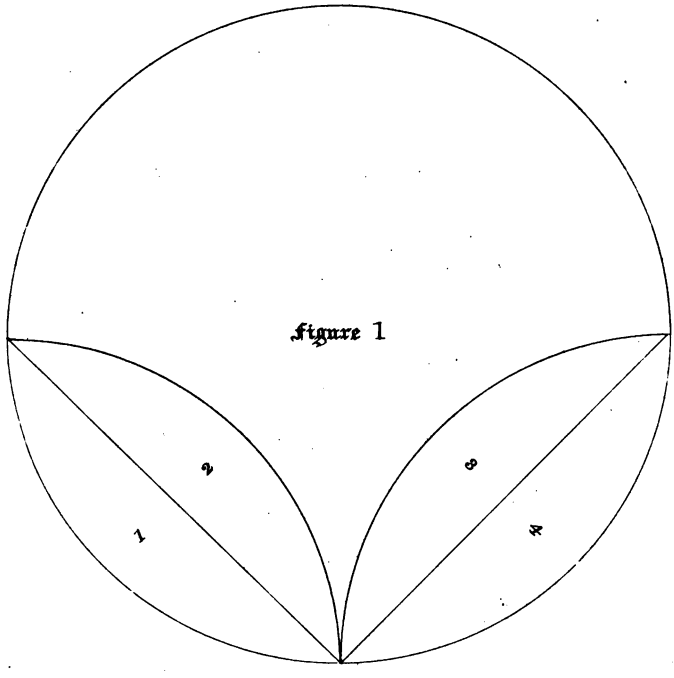


Figure 1

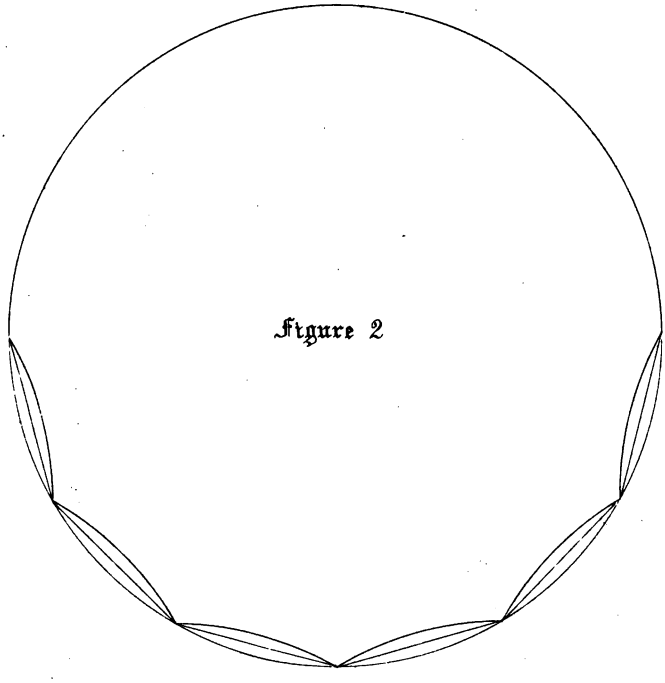
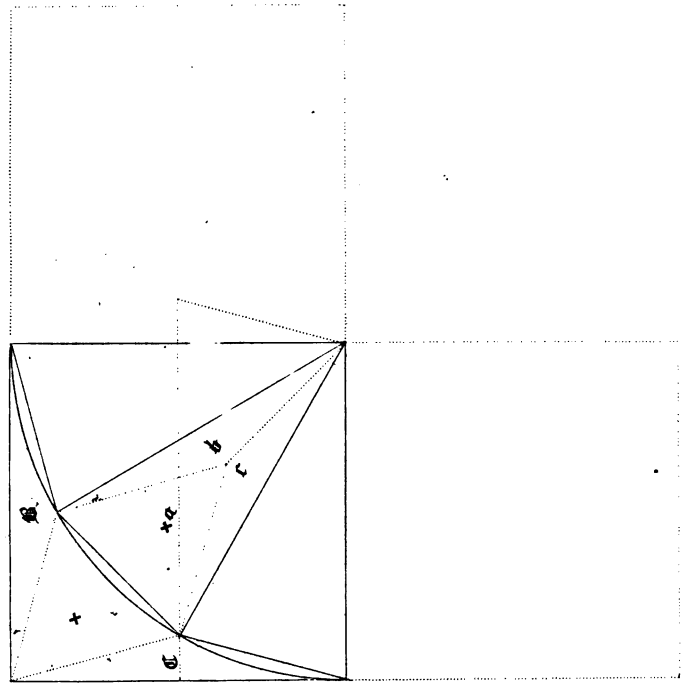


Figure 2

Figure 3



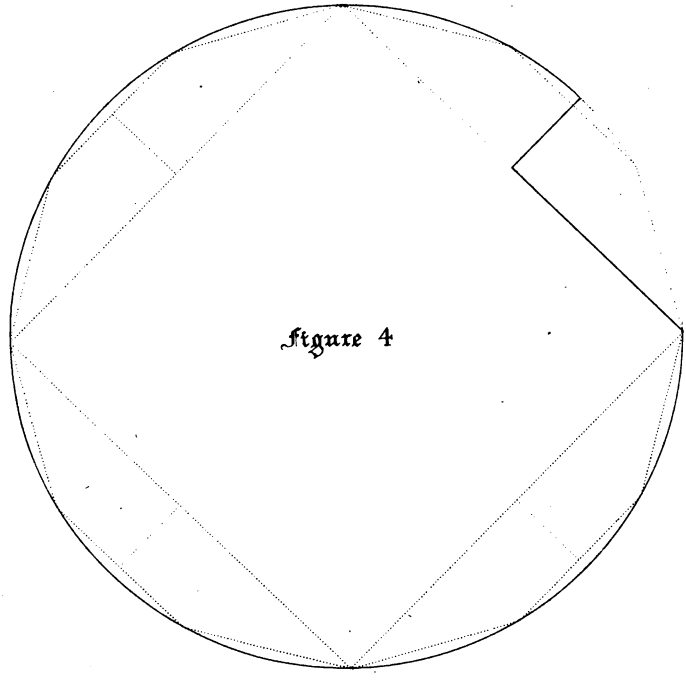


Figure 4

Figure 3. Within the quadrant there are 3, out of the 12, parts of the dodecagon. As equiangular figure X is equal to Xa, B is equal to b, and C is equal to c. Such square is, therefore, equal to 4 parts of the 12 of the dodecagon. Another proof. Project a line, dotted, from a point of the circumference at R, below the quadrant, equal to the radius, and connect such with centre of the circle. The dotted lines below the quadrant contain an area equal to $\frac{1}{2}$ of B. Therefore 2 parts of the 12 of the dodecagon are equal to half the square containing the quadrant. As this square is equal to $\frac{1}{2}$ the inscribed square, the inscribed square contains 8 of the 12 even parts of the dodecagon. The inscribed square and $\frac{1}{2}$ such square are therefore equal to the dodecagon.

Let us take the base of the inscribed square of the circle here at	700
	700
This multiplied by itself is equal to	490000
Divide by 2 and add product	245000
Area of dodecagon	735000
Divide the $\frac{1}{2}$ square or 245000 by 8 and add product . .	30625
Dodecagon and $\frac{1}{8}$th of $\frac{1}{2}$ square	765625

The base of inscribed square and $\frac{1}{4}$ of such base multiplied by the same should produce the above figures $700 + 175 = 875$

	875
	4375
	6125
	7000
Dodecagon and $\frac{1}{8}$th of $\frac{1}{2}$ square	765625

NOTE.—$877\frac{1}{2} \times 877\frac{1}{2}$ will within about half a dozen units square this circle.

In the vast consideration this subject has received, it has probably escaped observation that the base and $\frac{1}{4}$ base, of the inscribed square, multiplied by the same, should produce such an area as the dodecagon—and $\frac{1}{8}$th of the $\frac{1}{2}$ square that added to the inscribed square makes such figure. The light this throws upon the whole subject cannot be over-estimated. In the first place, it is obvious

this $\frac{1}{8}$th of such $\frac{1}{2}$ square includes—being over and above the dodecagon—so many of the 12 segments as go to complete the circle. How many and what parts of them?

That is the question. As this $\frac{1}{8}$th is an aliquot part of the inscribed square and an aliquot part of such $\frac{1}{2}$ square, a certain number of these 12 segments are probably, if not certainly, an aliquot part of such $\frac{1}{8}$th of the $\frac{1}{2}$ square above the area of the dodecagon. But what is more than merely remarkable, what appears to be almost wonderful, and a key to a postern gate of this mystery, is the fact that the multiplication of base and $\frac{1}{4}$ base, by base and $\frac{1}{4}$ base, as above, the first product—or first line of figures—is 4375, which in itself is nothing surprising, but when weighed and duly considered, as to the position in which it is found, and its relation to the geometrical figures considered, is of the utmost importance. In the first place it is infallible proof that such $\frac{1}{8}$th is a multiple of a certain number of these segments—and this particular $\frac{1}{8}$th above the dodecagon is only dealing with such segments—and because 4375 multiplied by 7 is equal to such $\frac{1}{8}$th of the $\frac{1}{2}$ square. This amount, or product, of the first figure so multiplying is $\frac{1}{7}$th of such $\frac{1}{8}$th of the $\frac{1}{2}$ of the inscribed square. Let us now endeavour to construct a figure equal to the dodecagon; and bound a certain number of segments—elusive beyond all imagination as they are to deal with—with as it were an impassable and adamantine frontier.

As the dodecagon is equal to the inscribed square and $\frac{1}{2}$ such square, it is manifest in Figure 4—bounded by $\frac{7}{8}$th of the circumference, and the two straight lines seen—we have such inscribed square and $\frac{7}{8}$th of the $\frac{1}{2}$ square that go to make up the dodecagon. It is manifest, also, we have in the dotted lines, marked off on this figure, $\frac{7}{8}$ths of the 12 segments. It is, of course, equally, manifest that if this figure be indeed equal to the dodecagon, then the $\frac{1}{2}$ segment—cut from the circle and required to complete it—contains the 12 segments beyond the dodecagon, and necessary to

complete the circle. Such segment, in the straight-sided portion of it, contains $\frac{1}{8}$th of the $\frac{1}{4}$ square, which should therefore be equal to $10\frac{1}{2}$ segments. The 1 segment and $\frac{1}{2}$ segment would therefore be $\frac{1}{7}$th of such $\frac{1}{8}$th, since 7 times 1 and a $\frac{1}{2}$ are $10\frac{1}{2}$. Let us see how this works out arithmetically.

The value of such $\frac{1}{8}$th of the $\frac{1}{4}$ square before mentioned we found to be 30625. This amount divided by 7 gives 4375. Arithmetic was good enough to put up a finger-post and point to these figures, nay, to present them to us as a perfect corner-stone, and a $\frac{1}{2}$, of the whole edifice, without which the wondrous halls of this Palace of Palaces, in all the symmetry of their perfect beauty, could not have been built. We therefore, gratefully, take advantage of it and them. The largest diamond, and of the first water, is found in the darkest of detritus, and most uninviting looking of earthy clays, and 'the strawberry underneath the nettle grows' as our Poet of Poets has said. This is possibly, nay probably, the correct value of such 1 segment and a $\frac{1}{2}$, for arithmetic in such matters makes no mistakes. But can we prove it? Mathematically, geometrically, and arithmetically we can. We can measure the straight-sided portion of Figure 4 exactly.

The inscribed square $700 \times 700 =$		490000
One eighth of the $\frac{1}{4}$ of such square is 30625.		
There are, as is seen, 7 such $\frac{1}{8}$ths. Therefore $30625 \times 7 =$ . .		214375
		704375
The value of 1 segment and $\frac{1}{2}$	4375	
Deduct $\frac{1}{2}$ or $\frac{1}{2}$ a segment .	$1458\frac{1}{3}$	
Value of 1 segment . .	$2916\frac{2}{3}$	
A glance at the figure shows us we have still $10\frac{1}{2}$ segments to add.		
Ten times $2916\frac{2}{3}$		$29166\frac{2}{3}$
One half of 1 segment (we also see remains to be added) as above .		$1458\frac{1}{3}$
The dodecagon, or its value, therefore, is this figure . . .		735000

The inscribed square is	490000
$\frac{1}{4}$ „ „	245000
	735000

This then (Figure 4) is the dodecagon, or its equal in value. It cannot be more: it cannot be less. It is not more: it is not less. We therefore know, now, for certain we can obtain the value of the 12 segments required to complete the circle, and exactly. The straight-sided portion of the ½ segment contains 10½ of the smaller segments, or this area, which is conclusive proof.

2916⅔ multiplied by 12	35000
Add to this the area of the dodecagon	735000
Area of circle	770000

The base and quarter base of inscribed square, multiplied by the same, give the area of the circle exactly, less the area of 1 segment and a ½ segment. The area of 1 segment and a ½ is given exactly, in the result of the first unit multiplying. As we know therefore, the ⅛th of the ½ square is over and above the area of the dodecagon, we also know this ⅛th of the ½ square is equivalent in area to the 10½ segments over and above the dodecagon. The ⅛th of the ½ square is therefore a multiple of the 1 segment and ½ segment.

Let us look at the result in a larger circle—one a million times the area of the one above—checking the result by two modes of calculation. Take the base of the inscribed square at 700000. Then

700000	
700000	
490000000000	square
245000000000	½ square
735000000000	dodecagon
30625000000	⅛th of ½ square
4375000000	⅛th of ⅛th
Area of circle 770000000000	

Base and ¼ base and fraction	875000¼
	875000¼
	4375000000
	6125000
	7000000
	765625000000
	2187500000
	2187500000
Area of circle	770000000000

In the second of the above the 5 increases the figures, the fraction's product beginning equally. Let us now take another circle, the *diameter* of which we will say is 700000. This multiplied by the radius will produce the inscribed square.

	700000
	350000
	350000000000
	2100000
	245000000000
Add ½ to produce dodecagon	122500000000
Add ⅛ of ½ square	15312500000
Add ¼ of ⅛ „	2187500000
Circle	385000000000

According with all the premisses, the diameter should be to the circumference as 1 to 3⅐. Let us check the above circle's area so ascertained as above. The diameter is—

	700000
	3⅐
	2100000
⅐ . . .	100000
	⅐)2200000
½ circumference	1100000
multiply by radius	350000
	550000000000
	330000000000
Circle's area	385000000000

As this is a rectangle 1100000 in length, and 350000 in

breadth, and the 1100000 is, exactly, ½ the circumference, this proves the 1st proposition that a curved line is equivalent to a straight one of the same length. The area of the last circle noticed, we have seen is 385000000000. Let us add a definite proportion to this but an unknown quantity, viz. $\frac{3}{11}$

	385000000000
	35000000000
	35000000000
	35000000000
Circle's area	490000000000

From what has been said before and proved, the 4 segments that bound the inscribed square are each $\frac{1}{11}$th of the circle. The circle to the square in which it is inscribed is as 11 to 14. Having composed a circle as above, containing an area of 490000000000, let us deduct from it $\frac{4}{11}$ths, which should therefore produce its inscribed square.

$\frac{4}{11}$ 490000000000

445454545451$\frac{8}{11}$

4

178181818181$\frac{9}{11}$

490000000000

178181818181$\frac{8}{11}$

311818181818$\frac{3}{11}$

inscribed square	311818181818$\frac{3}{11}$
½ „ „	155909090909$\frac{1}{11}$
dodecagon	467727272727$\frac{4}{11}$*
⅛th of ½ square	19488636363$\frac{8}{88}$ $\frac{1}{88}$
⅐th of ⅛th of ½ square	2784090909$\frac{6}{616}$ $\frac{1}{616}$
circle area in square	490000000000

700000 × 700000 is the square of this circle.

So a circle larger than the one of which the diameter is 700000 by $\frac{3}{11}$ths of its area, is exactly squared. But it may be said of this circle we know no more of it than its area. We do not know its diameter, the base of its inscribed square, or any further particulars than its inscribed square, its dodecagon, the ⅛th of the ½ square, and the ⅐th of such

* These fractions reduced are $\frac{1}{616}$, $\frac{44}{616}$, $\frac{448}{616}$, $\frac{44}{616}$, $\frac{7}{616}$, $\frac{1}{616}$.

$\frac{1}{8}$th, namely the area of 1 segment and a $\frac{1}{2}$ segment, and its total area. The answer to this simply is, these are the fullest particulars, for its area and these details are given exactly; and the diameter, as nearly as it can be expressed in plain figures of arithmetical language is $789707\frac{5}{113}$. The square of such circle is therefore $700000 \times 700000 = 490000000000$.

As before said, it is known that Archimedes made use of a polygon of 96 sides. It is interesting to try to discover, and, perhaps, elucidate a line of Greek thought. It may be safely considered certain he selected this after careful consideration. Did he compare this polygon with the one of 12 sides? Probably he did. He would no doubt find that 12 segments of the 96 would correspond, as they do, with the 1 segment and $\frac{1}{2}$ segment taken—and given—as the test and standard of value here. 12 such segments being an eighth of 96, and these 12 would be, and are, an aliquot part of the straight-sided figure in the larger 1 segment and $\frac{1}{2}$, containing an area equal to 96 less 12, or 84 of them, because 12 of these smaller segments are $\frac{1}{7}$th of 84. And so on endlessly with 8 times 96 and 8 times the product each time, the $\frac{1}{8}$th of the smaller segments being $\frac{1}{7}$th of the straight-sided figure of the next largest. Carrying this from 12 multiplied by 8, and the product and products so on by 8 for 10 times only, the product will be 12884901888 more than 12 billions 884 millions of segments. The segments so reduced, as no one can detect a curve in $\frac{1}{2}$ an inch, even in so small a circle as the earth's circumference, appear to dissolve, as it were, into a different marshalling of atoms, the next line that can be drawn within the perimeter of a true circumference, or circle, being, perhaps, only one point, or atom, less than the outer one. This atom added will make the curved line equal to a straight one as it is, as we have seen and proved. But how hopeless, and apparently how unphilosophical, to attempt to reach this by a standard too short, or too long, or both.

Let us consider and measure a larger circle than any considered, in two different ways, checking it beyond the possibility of imaginary error.

The apex of the solar way is not agreed upon exactly; but the sun is apparently revolving round his own sun in the constellation of Pleiades. Assume the distance of such sun at about ½ a million more of the semi-diameters of the earth's orbit than the star Capella, or say 500000000 semi-diameters of the earth's orbit.

92000000 × 500000000 = 46000000000000000

92000000 × 500000000

Light, flashing round the world 6 times in a second, would reach here thence in 81 years. This would be the radius of a circle described by our sun and this earth round the sun's sun. Double this for the diameter of such orbit.

	9200000000000000
Multiply by radius to obtain square	46000000000000000
	552000000000000000000000000000000
	368000000000000000
inscribed square	423200000000000000000000000000000
½ „ „	211600000000000000000000000000000
⅛th „ „	26450000000000000000000000000000
⅐th „ „	3778571428571428571428571428571⅜
Circle's area	6650285714285714285714285714285714⅜

Another way—

	Fraction.
9200000000000000	46000000000000000
3⅐	3
276000000000000000	⅓)138000000000000000
13142857142857142⅜	19714285714285714⅜
½)28914285714285714⅜	
14457142857142857⅛	
46000000000000000	
867428571428571426000000000000000	
578285714285714284000000000000000	
19714285714285714⅜	
Circle's area 6650285714285714285714285714285714⅜	Sun's orbit.

These are miles, and the one sum checks the other, and this is the exact area of such a circle as we have supposed for the sun's orbit. If these be reduced to yards, feet, inches, and the base of the 8000 *milliards of milliards* spoken of before, you have exactly the number of atoms of matter, assuming such estimate of its divisibility to be correct, in such circle. How many atoms of ether, of the force producing light, and the force called gravity, acting upon and through ether, we cannot say; but this circle measures all.

The only idea we can form of it is that light would take about 500 years to travel round it.

The 12 segments, beyond and above the dodecagon, contain all the atoms that can be comprised within such circumference, or line, without breadth. The $\frac{1}{8}$ segment contains the 12, or their equivalent, and therefore contains them all.

It will, perhaps, be remembered that where units and ciphers multiply the same, in the multiplication of base and $\frac{1}{4}$ base, by base and $\frac{1}{4}$ base, of inscribed square, the product of the first unit multiplying gives the area of $\frac{1}{8}$th of the $\frac{1}{8}$ of the inscribed square. This has now to be shown, together with another circle exactly squared. The circle before added to by $\frac{5}{11}$ths might be objected to as one to correspond, but really given to prove the circle to be, to the square in which such circle is inscribed, as 11 to 14 exactly. Take the base of the inscribed square of another circle at 1000000.

	1000000 1000000	Base and $\frac{1}{8}$ base by same and fraction	$1250000\frac{1}{8}$ $1250000\frac{1}{8}$
Square	1000000000000		625000000000
$\frac{1}{2}$ „	500000000000		250000
Dodecagon	1500000000000		1250000
$\frac{1}{8}$th of $\frac{1}{2}$ square	625000000000		1562500000000
$\frac{1}{7}$th of this	$89285714284\frac{4}{7}$		$446428571\frac{3}{7}$ $446428571\frac{3}{7}$
Area of circle	$1571428571428\frac{4}{7}$	Area of circle	$1571428571428\frac{4}{7}$

It is here seen in the multiplication of base by base of

square, that 625000000006 is the $\frac{1}{8}$th of the $\frac{1}{2}$ of the inscribed square; and that in the multiplication of base and $\frac{1}{4}$ base and fraction by base and $\frac{1}{4}$ base and fraction the '5' or first unit multiplying produces the same. The next two figures, here, produce the dodecagon. The $\frac{1}{25}$th must be taken into 125 and nine ciphers to put the product in the right place, and to produce it exactly.

In the first circle considered, the base of its inscribed square being 700, the value of one segment was found to be 2916$\frac{2}{3}$. The 12th proposition states 'The circle contains an area equal to 264 of these segments.' This has now to be proved—and which completes the proofs of the 12 propositions—the others having now all been proved. The area of this circle was found to be 770000, 2916$\frac{2}{3}$ × 264 should produce this—

$$\begin{array}{r} 2916\frac{2}{3} \\ 264 \\ \hline 11840 \\ 17496 \\ 5832 \\ \hline 770000 \end{array}$$

Producing 4 such segments (as in addition to the inscribed square complete the circle) within such square, it follows as a corollary on what has been said and seen, that the space left within the square is equal to 3 such segments, and as this space is manifestly equal to the 4 spaces that together make up the difference between the circle and the square in which it is inscribed, such 3 segments, each containing as we have seen 24 of the smaller segments of the value of 2916$\frac{2}{3}$ each, should, added to the above circle's area, 770000, produce the square in which it is inscribed. Let us see 2916$\frac{2}{3}$ × 72—

$$\begin{array}{r} 2916\frac{2}{3} \\ 72 \\ \hline 5880 \\ 20412 \\ \hline 210000 \end{array}$$ This added to area 770000 = 980000,

which should be the value of the square in which such circle is inscribed. Look at Figure 4. It is manifest, looking at the base of the inscribed square there, that such base multiplied by ½ such base will give the value of a square one-fourth of the area of the square in which the circle is inscribed.

700 × 350
350
35000
2100
245000
This multiplied by 4 is such square 4
980000

It follows that the 336th part of any square multiplied by 264 will produce the area of that square's circle within it.

The mere squaring of the circle, as it is called, is a matter of little or no moment in itself, as nothing depends upon it; and though some objection may be taken that some of the circles are not squared by the multiplication of the same array of units by the same, the answer to this is, the correct result cannot so be attained, as the transposition, in the last figure, of a larger unit for a smaller, or the reverse, makes this greatly too much, or too little; and as the method adopted gives the area exactly, it may not be objected to.

The all-important matter, of course, is finally to establish and prove the ratio of the diameter to the circumference, and find the correct area, which is done, and in each case, here, exactly.

There are subtle and intrinsic harmonies, combinations, proportions, and laws, as in language, chemistry, and music, so in geometry and arithmetic, that have an initiation and originality far beyond the mere invention and imagination of man; and these are as fixed and inviolable as they are beautiful and perfect. And no one, perhaps, though disposed to be prejudiced, will be inclined seriously to believe

or aver that this rule, giving the value of the dodecagon, and particularly and especially of the $\frac{1}{8}$th of the $\frac{1}{2}$ square, in addition, is less precise and exact in its declaration of the exact value of this, and of what is the same thing—or same in area—the $10\frac{1}{2}$ segments above the dodecagon, than it is absolutely exact in the area of such $1\frac{1}{2}$ segment as alone remains to complete the circle, which is particularly and exactly specified in the product—in the multiplication of base and $\frac{1}{4}$ base by the same—of the first unit multiplying.

It will be remembered this rule does not exist to give the value of the $\frac{1}{8}$th of the $\frac{1}{2}$ square, since this is found otherwise, and is seen as a distinct figure and aliquot part of the dodecagon itself, but to give—as accurately as the diameter multiplied by the radius gives the exact area of the inscribed square—the exact value of the dodecagon proper, and a certain number or definite proportion of the 12 segments beyond it—that is to say, $\frac{7}{8}$ths of them, or $10\frac{1}{2}$.

To suppose for a moment that in so remarkable and indeed wonderful a rule, one single atom even is omitted from, or added to, the true and exact area of, so evidently selected and singularly chosen, a figure as $\frac{1}{2}$ a segment, would be a *reductio ad absurdam,* and a perversion of the understanding; the more so as the $\frac{1}{8}$th of the $\frac{1}{2}$ square is a multiple of $1\frac{1}{2}$ segment, as all the above proofs show, and which are so corroborated by hundreds of corollary proofs.

To attempt to shake a superstructure, built upon such a basis, by a formula framed upon erroneous standards of outer and inner polygons, and confessedly inadequate, is hopeless; and would seem, indeed, to involve a negation, or contradiction, of the first axiom the mathematician sets out with, that a point is 'without length, breadth, and thickness,' and, therefore, cannot be attained to or reached by a thousand places of decimals, or as many as there are letters in the pages of this pamphlet.

The circle is therefore squared, in more ways than one,

in many ways. The exact relationship of diameter to circumference proved, and its capacity and importance, as an instrument of measure, fully shown. The long, almost interminable procession of 7ths, have presented themselves as an avenue of approach. The one segment has constituted as it were the portals, or main entrance, to this well guarded and really wonderful region of unending and ever new and varied beauties, and the 'Half Segment' recognised by arithmetic, and especially and particularly incorporated, and indeed held up to view, in one of its most beautiful rules, has been the Golden Key to a figure which mathematically, geometrically, and arithmetically equal to the dodecagon, or as it may be said—the dodecagon in other guise—has acted as the spectroscope in its sphere, and brought to light the nature, hidden worth, and exact value of all these segments of the circle and the circle's exact area.

As earthly things, in many ways, are but symbols of heavenly ones, it would not be becoming in one, professing to be a Christian, to omit to notice how all this, in the principal numbers used, draws attention to the far-distant reaching days, or generations, of the earth's creation, through all time from the beginning, prepared, furnished, and fitted by the Almighty, with everything needful, useful, and most beautiful, for His chief care and delight and principal work—man, with its ages of 7ths and seventh days rest, so highly sanctioned and therefore so obligatory upon and needful to us. This conjunction of 7 with 3, and with 12, almost necessarily reminds us of this, of the Holy Triune Deity, of the foundations upon which He has built the world, and His church; of the 12 months of the year, the 12 jewels of the breastplate, the 12 signs of the Zodiac: and to this consideration is added more weight, and which, in itself, removes it far from the fanciful, when we know that He Himself who has made, and upholds all, has said to you and to me, and to each one of us, of those seven stars,

or suns, which under His feet, perhaps, support us all; and as surely as He ever spoke any other word upon earth, 'Canst thou bind the sweet influences of Pleiades?'

I conclude, therefore, with praise and great thankfulness to Him who has enabled me to accomplish, and make known, what has never been so fully made known before.

ALEX. BURNES ANDERSON.

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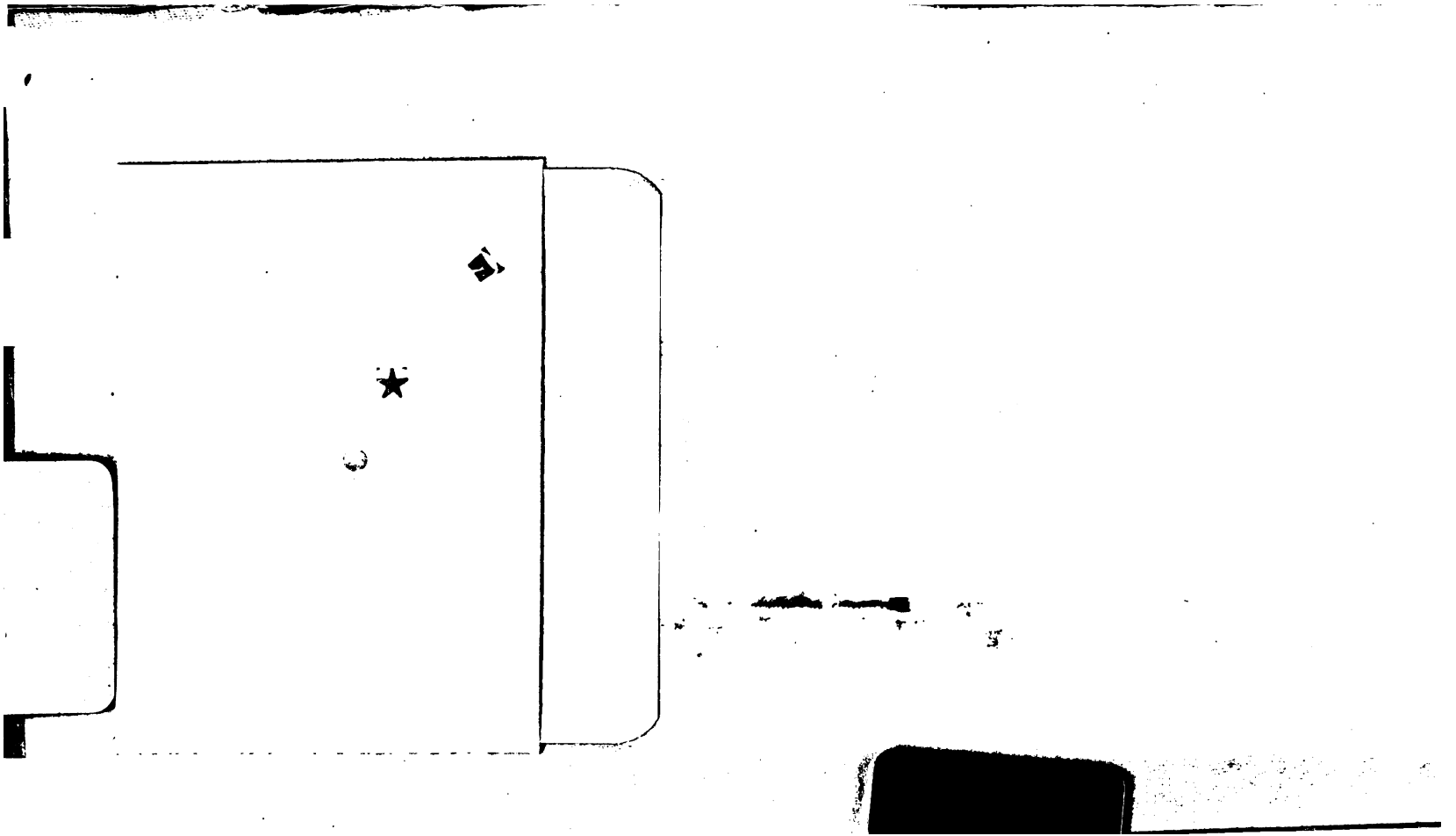

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